

積分おさらいテスト NO2

学籍番号

氏名

(44) オイラーの公式を用いて三角関数の加法定理 $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$ を導け.

$$\begin{aligned} \varepsilon^{j(\alpha+\beta)} &= \cos(\alpha+\beta) + j \sin(\alpha+\beta) \dots \textcircled{1} \\ \varepsilon^{j(\alpha+\beta)} &= \varepsilon^{j\alpha} \cdot \varepsilon^{j\beta} = (\cos\alpha + j \sin\alpha)(\cos\beta + j \sin\beta) \\ &= (\cos\alpha \cos\beta - \sin\alpha \sin\beta) + j(\sin\alpha \cos\beta + \cos\alpha \sin\beta) \end{aligned}$$

$\dots \textcircled{2}$
①, ②より

$$\begin{cases} \sin(\alpha+\beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta \\ \cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta \end{cases}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

(45) オイラーの公式を用いて倍角の公式 $\sin 2\alpha$, $\cos 2\alpha$ を求め, 半角の公式 $\sin^2(\alpha/2)$, $\cos^2(\alpha/2)$ を求めよ.

$$\begin{aligned} \varepsilon^{j2\alpha} &= \cos 2\alpha + j \sin 2\alpha \dots \textcircled{1} \\ \varepsilon^{j2\alpha} &= (\cos\alpha + j \sin\alpha)^2 \\ &= (\cos^2\alpha - \sin^2\alpha) + j(2 \sin\alpha \cos\alpha) \dots \textcircled{2} \end{aligned}$$

①, ②より

$$\begin{cases} \sin 2\alpha = 2 \sin\alpha \cos\alpha \\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha \end{cases}$$

$$\begin{aligned} \sin^2\alpha + \cos^2\alpha &= 1 \quad \text{より} \\ \cos 2\alpha &= 1 - 2 \sin^2\alpha = 2 \cos^2\alpha - 1 \\ \alpha &\rightarrow \alpha/2 \quad \text{と置き換えると} \end{aligned}$$

$$\sin^2 \frac{\alpha}{2} = \frac{1 - \cos\alpha}{2}$$

$$\cos^2 \frac{\alpha}{2} = \frac{1 + \cos\alpha}{2}$$

(46) 加法定理の結果を用いて, 積和の公式 $\sin\alpha \cos\beta$, $\cos\alpha \sin\beta$, $\cos\alpha \cos\beta$, $\sin\alpha \sin\beta$ を導け.

$$\begin{cases} \sin(\alpha+\beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta \quad \dots \textcircled{1} \\ \sin(\alpha-\beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta \quad \dots \textcircled{2} \end{cases}$$

①+②より

$$\sin\alpha \cos\beta = \frac{1}{2} \{ \sin(\alpha+\beta) + \sin(\alpha-\beta) \}$$

①-②より

$$\cos\alpha \sin\beta = \frac{1}{2} \{ \sin(\alpha+\beta) - \sin(\alpha-\beta) \}$$

$$\begin{cases} \cos(\alpha+\beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta \quad \dots \textcircled{3} \\ \cos(\alpha-\beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta \quad \dots \textcircled{4} \end{cases}$$

③+④より

$$\cos\alpha \cos\beta = \frac{1}{2} \{ \cos(\alpha+\beta) + \cos(\alpha-\beta) \}$$

③-④より

$$\sin\alpha \sin\beta = -\frac{1}{2} \{ \cos(\alpha+\beta) - \cos(\alpha-\beta) \}$$

$$(47) \int \sin^2 x dx$$

$$\frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x dx = \frac{1}{2}x - \frac{1}{4} \sin 2x + C$$

$$(48) \int \sin 2x \cos 3x dx$$

$$\frac{1}{2} \int \sin 5x dx - \frac{1}{2} \int \sin x dx = -\frac{1}{10} \cos 5x + \frac{1}{2} \cos x + C$$